# Self-excited walking of a biped on various geometrical surfaces 

Alinjar Dan ${ }^{1}$, S.K.Saha ${ }^{2}$, K.RamaKrishna ${ }^{3}$<br>${ }^{1}$ Research Scholar of Mechanical Engineering Indian Institute of Technology Delhi IIT Campus, Hauz Khas, New Delhi, Delhi , 110016, India mez188288@mech.iitd.ac.in<br>${ }^{2}$ Faculty of Mechanical Engineering Indian Institute of Technology Delhi<br>IIT Campus, Hauz Khas, New Delhi, Delhi , 110016, India saha@mech.iitd.ac.in<br>${ }^{3}$ Faculty of Mechanical Engineering<br>Indian Institute of Technology Delhi<br>IIT Campus, Hauz Khas, New Delhi, Delhi , 110016, India<br>rkk@mech.iitd.ac.in

## EXTENDED ABSTRACT

## 1 Introduction

Locomotion of a biped has been an area of interest among researchers for a long period. A biped robot is an underactuated system, therefore it is challenging to ensure its stable locomotion, especially on non-flat surfaces. Some of the noted works in this area are available at [1]-[3]. In the current work, dynamic modelling of a biped is performed and a Proportional-Derivative (PD) control is applied to it. Stable locomotion of the biped on different surfaces is achieved by varying the proportional and derivative gain values.

## 2 Dynamic Modelling



Figure 1: Kinematic architecture of a self excited biped
In this work, biped is modelled as a 3 Degree of Freedom ( DoF ) serial chain system (Figure 1). This mechanism of biped is inspired by [1]. The Stance leg is assumed to be straight and has 1 DOF, whereas, the swing leg consists of 2 DOFs. In addition, the following assumptions are considered for simplification. (a). All links are rigid and cylindrical. (b). The Centre of gravity of each link lies in the geometric center of the respective link. (c). Legs do not have feet.

The Euler-Lagrange methodology is used for the dynamic modeling of the biped. Only hip joint is actuated (Figure 1). Equations of Motion (EoM) of the biped are given by:

$$
\left[\begin{array}{lll}
M_{11} & M_{12} & M_{13}  \tag{1}\\
M_{12} & M_{22} & M_{23} \\
M_{13} & M_{23} & M_{33}
\end{array}\right]\left[\begin{array}{c}
\ddot{\theta}_{1} \\
\ddot{\theta}_{2} \\
\ddot{\theta}_{3}
\end{array}\right]+\left[\begin{array}{ccc}
0 & C_{12} \dot{\theta}_{2} & C_{13} \dot{\theta}_{3} \\
-C_{12} \dot{\theta}_{1} & G & C_{23} \dot{\theta}_{3} \\
-C_{13} \dot{\theta}_{1} & -C_{23} \dot{\theta}_{2}-G & G
\end{array}\right]\left[\begin{array}{l}
\dot{\theta}_{1} \\
\dot{\theta}_{2} \\
\dot{\theta}_{3}
\end{array}\right]+\left[\begin{array}{l}
K_{1} \\
K_{2} \\
K_{3}
\end{array}\right]=\left[\begin{array}{c}
0 \\
K_{p} \theta_{3}+K_{d} \dot{\theta}_{3} \\
0
\end{array}\right]
$$

where $K_{p}$, and $K_{d}$ are Proportional and Derivative gains respectively. The other matrix elements are given below.
$M_{11}=I_{1}+m_{1} a_{1}^{2}+m_{2} l_{1}^{2}+m_{3} l_{1}^{2}, M_{12}=\left(m_{2} a_{2}+m_{3} l_{2}\right) l_{1} \cos \left(\theta_{2}-\theta_{1}\right), M_{13}=m_{3} a_{3} l_{1} \cos \left(\theta_{3}-\theta_{1}\right), M_{22}=I_{2}+m_{2} a_{2}^{2}+m_{3} l_{3}^{2}, M_{23}=$ $m_{3} a_{3} l_{2} \cos \left(\theta_{3}-\theta_{2}\right), M_{33}=I_{3}+m_{3} a_{3}^{2}, C_{12}=-\left(m_{2} a_{2}+m_{3} l_{2}\right) l_{1} \sin \left(\theta_{2}-\theta_{1}\right), C_{13}=-m_{3} a_{3} l_{1} \sin \left(\theta_{3}-\theta_{1}\right), C_{23}=-m_{3} a_{3} l_{2} \sin \left(\theta_{3}-\right.$ $\left.\theta_{2}\right), K_{1}=\left(m_{1} a_{1}+m_{2} l_{1}+m_{3} l_{1}\right) g \sin \theta_{1}, K_{2}=\left(m_{2} a_{2}+m_{3} l_{2}\right) g \sin \theta_{2}, K_{3}=m_{3} l_{3} g \sin \theta_{3}, G=$ viscous damping coefficient.
Here $m_{i}, l_{i}$, and $a_{i}$ represent the mass, the length, and the distance from the origin to the centre of mass of the $i$-th link respectively. The term $I_{i}$ represents the mass moment of inertia of the $i$-th link w.r.t axis perpendicular to the plane of walking and passing through the origin of the $i$-th link.

Two impacts take place in each cycle of the biped locomotion. One of them is knee impact which occurs at the knee joint of the swing leg due to the presence of a knee locking mechanism at the knee joint (virtual for simulation). Knee locking prevents hyperextension of the shank, i.e., $\theta_{3} \leq \theta_{2}$ always. The second impact is the ground impact, i.e., the impact between the feet of the swing leg and the ground. Both impacts are considered instantaneous and they are modelled using the law for the conservation of angular momentum at the point of impact.

## 3 Results

In this section, the results of biped locomotion on different surfaces are presented. The different values of $K_{p}$ and $K_{d}$ for each surface are listed in Table 1. The limit cycles for those walking at the last step of walking (10th step) are shown in Fig. 2. Closed limit cycles imply that the generated gait pattern of the biped is periodic and stable. The videoes of the same simulations are attached at this link- shorturl.at/jruC3. In all the simulation videos, the Zero Moment Point (ZMP) of the biped is marked as a red circle.

Table 1: Proportional ( $K_{p}$ ) and Derivative ( $K_{d}$ ) gains

| Surface | $K_{p}$ | $K_{d}$ |
| :--- | :---: | :--- |
| Flat | 6 | 0 |
| Inclined (Incline $\left.=2^{\circ}\right)$ | 6 | 1 |
| Inclined $\left(\right.$ Incline $\left.=4^{\circ}\right)$ | 6 | 1.2 |
| Circular $($ RoC $=8 m)$ | 6 | 1.4 |


(a) Limit Cycle for walking on a flat surface

(b) Limit Cycle for walking on an inclined sur-
face of 2-degree incline

(c) Limit Cycle for walking on an inclined surface of 4-degree incline

(d) Limit Cycle for walking on a circular surface

Figure 2: Limit Cycle of biped at the 10th cycle of walking on different surfaces

## References

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